B.Sc. Semester-IV Examination, 2022-23

PHYSICS [Honours]

Course ID : 42411 **Course Code : SH/PHS/401/C-8(T8)**

Course Title : Mathematical Physics-III

Time : 1 Hour 15 Minutes Full Marks : 25

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

SECTION–I

1. Answer any **five** questions: 1×5=5

   a) State the Change of Scale property of Laplace transform.

   b) What is the Fourier transform of $\delta(t-a)$ where a is a constant?

   c) What is the nature of a Gaussian function after Fourier transform?

   d) State Fourier integral theorem.

   e) Find Laplace transform of $t\cos t$.

   f) State Cayley-Hamilton theorem.

   g) What is the norm of vector (1, 0, 1)?

[Turn Over]

SECTION–II

2. Answer any **two** questions: 5×2=10

   a) State and prove Convolution theorem of Fourier transform. 1+4

   b) Generate an orthonormal set from the LI set (2, 0, 1); (2, 1, 3); (4, 1, 2) in R^3.

   c) Solve the following coupled differential equation:

$$3\frac{dx}{dt}-y=2t$$

$$\frac{dx}{dt}+\frac{dy}{dt}-y=0$$

   with the condition x(0) = y(0) = 0.

   d) Show that $\int\frac{\cos\lambda x}{1+\lambda^2}d\lambda=\frac{\pi}{2}e^{-x}$ for $x\geq 0$.

SECTION–III

3. Answer any **one** question: 10×1=10

   a) i) A resistance R in series with inductance L is connected with e.m.f. $\varepsilon(t)$. The current is given by

$$L\frac{di}{dt}+Ri=\varepsilon(t).$$

If the switch is connected at t=0 and disconnected at t=a, using Laplace transformation find the current i in terms of t.

ii) Find the Fourier Sine transform of $e^{-|x|}$.

Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx$. 5+(2+3)

b) i) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ using Fourier Transform under the condition

$$\begin{aligned} u &= 0, \quad at \;\; x = 0 \\ &= 1, \quad 0 < x < 1 \\ &= 0 \qquad x \geq 1 \end{aligned}$$

when t=0 and u is bounded.

ii) Find the characteristic equation of the symmetric matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

Apply Cayley-Hamilton theorem to obtain A^{-1}. 6+4

——————